

# Isospin violating decays of positive parity $B_s$ mesons in $\text{HM}\chi\text{PT}$

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Recent lattice QCD results suggest that the masses of the first two positive parity  $B_s$  mesons lie below the  $BK$  threshold, similar to the case of  $D_{s0}^*(2317)^+$  and  $D_{s1}(2460)^+$  mesons. The mass spectrum of  $B_s$  mesons seems to follow pattern of  $D_s$  mass spectrum. As in the case of charmed mesons, the structure of positive parity  $B_s$  mesons is very intriguing. To shed more light on this issue, we investigate strong isospin violating decays  $B_s(0^+) \rightarrow B_s^0\pi^0$ ,  $B_s(1^+) \rightarrow B_s^{*0}\pi^0$  and  $B_s(1^+) \rightarrow B_s^0\pi\pi$  within heavy meson chiral perturbation theory. The two body decay amplitude arises at the tree level and we show that the loop corrections give significant contributions. On the other hand, in the case of three body decay  $B_s(1^+) \rightarrow B_s^0\pi\pi$  amplitude occurs only at the loop level. We find that the decay widths for these decays are:  $\Gamma(B_s(1^+) \rightarrow B_s^0\pi\pi) \sim 10^{-3} \text{ keV}$  and  $\Gamma(B_s(0^+) \rightarrow B_s^0\pi^0) \leq 55 \text{ keV}$ ,  $\Gamma(B_s(1^+) \rightarrow B_s^{*0}\pi^0) \leq 50 \text{ keV}$ . More precise knowledge of the coupling constant describing the interaction of positive and negative parity heavy mesons with light pseudoscalar mesons would help to increase accuracy of our calculation.

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## I. INTRODUCTION

Two positive parity mesons  $B_s$  states: the  $J^P = 1^+$  state  $B_{s1}(5830)^0$  and the  $J^P = 2^+$  state  $B_{s2}^*(5840)^0$  were observed by the CDF and LHCb collaborations [1–4]. Recent lattice results [5], as well as previous works of the authors [6–13], have indicated that the observed states are most likely members of the  $(1^+, 2^+)$  doublet. This indicates that the positive parity doublet of  $B_s$  states  $(0^+, 1^+)$  is still unobserved. Above mentioned studies also suggest that the  $(0^+, 1^+)$  doublet of  $B_s$  states might have masses below the  $BK$  and  $B^*K$  thresholds. However, some relativistic quark models analysis [14–16] suggested that masses of the  $(0^+, 1^+)$  doublet  $B_s$  states should be above the  $BK$  and  $B^*K$  thresholds. This reminds strongly of the "history" of establishing charm meson spectrum in which detection of positive parity states below  $DK$  threshold was not predicted by the quark models. After the observation of  $D_{s0}^*(2317)$  and  $D_{s1}^*(2460)$  charmed mesons, we faced a long-lasting dilemma on the structure of  $D_s(0^+, 1^+)$ . The issue is whether the  $D_{s0}^*(2317)$ , and  $D_{s1}^*(2460)$  are  $\bar{q}q$  states or more exotic compounds [17]. It was already suggested by authors of [17], that study of strong and radiative decay modes of positive parity  $D_s$  states might help in differentiating between these scenarios. Since, both systems of positive parity states  $D_s$  and  $B_s$  are rather similar, the systematic analyses of strong and radiative decay dynamics of  $B_s$  mesons would help in clarifying their structure. In our study we rely on the results of lattice calculations presented in Ref. [5]. These authors determined the spectrum of  $B_s$   $1P$  states and they found that masses of  $B_{s1}(5830)^0$  and  $B_{s2}^*(5840)^0$  agree very well with experimental results [1–4]. They predicted also existence of the spin zero positive parity state ( $J^P = 0^+$ ) with the mass  $m_{B_{s0}} = 5.711(13)(19)$  GeV and the state  $J^P = 1^+$  with the mass  $m_{B_{s0}} = 5.750(17)(19)$  GeV. Both states have masses below  $BK$  and  $B^*K$  threshold. This immediately indicates that both these states can decay strongly if isospin is violated. Motivated by the result of lattice calculation and relying on our findings in the appropriate charm sector [18], we determine partial decay widths of both meson states to the final state containing one or two pions:  $B_s(0^+) \rightarrow B_s^0\pi^0$ ,  $B_s(1^+) \rightarrow B_s^0\pi^0$  and  $B_s(1^+) \rightarrow B_s^0\pi\pi$ .

Studies of these decays were performed already by [11–13, 19, 20]. Authors of [20] assumed that the positive parity  $0^+$  and  $1^+$   $B_s$  states have a structure of  $BK$  molecules, accounting for the similarity with  $D_s$ , and they suggest that they are rather narrow states with partial decay widths about 50 – 60 keV. On the other hand, authors of [11–13, 19] used different approach based on the assumption that the decays proceed through the channels  $B_s(0^+) \rightarrow B_s^0\eta \rightarrow B_s^0\pi^0$  and  $B_s(1^+) \rightarrow B_s^0\eta \rightarrow B_s^0\pi^0$  with the help of  $\eta - \pi$  mixing and predicted the partial decay widths in a range of 10 – 40 GeV. As already discussed in [18, 21–23] chiral loop corrections play an important role in strong decays of  $D_s$  positive parity states and their contribution to the strong decay modes can be as large as the effect of  $\eta - \pi$  mixing. Since  $\pi$  and  $\pi\pi$  in the final state of these decays are having very small momenta, both decay modes are ideal to use heavy meson chiral perturbation theory (HM $\chi$ PT).

In this paper, we determine the isospin violating decay amplitudes of positive parity  $B_s$  mesons, members of the  $(0^+, 1^+)$  doublet, using HM $\chi$ PT. For two-body decays, there is a tree-level contribution to decay amplitude arising from the  $\eta - \pi$  mixing and loop contribution which is the divergent. The divergent loop contribution requires the regularisation by the counter-terms. On the other hand, in the isospin violating two body decays of  $D_{s0}(2317)$  and  $D_{s1}(2460)$  mesons, chiral loops contribute significantly [18]. This was indicated already in Ref. [24] within different framework in which only part of the loop contributions are included in the decay amplitudes of  $D_{s0}(2317)$  and  $D_{s1}(2460)$ . As we pointed out in [18], the isospin violating three body decay amplitude can arise at the loop level only within HM $\chi$ PT. These loop contributions are then finite. In the case of charm decays, the ratio of the decay widths for  $D_{s1}(2460)^+ \rightarrow D_s^{*+}\pi^0$  and  $D_{s1}(2460)^+ \rightarrow D_s^+\pi^+\pi^-$  is known experimentally. From this ratio we were able to constrain the finite size of the counter-terms necessary to regularise two body decay amplitude  $D_{s1}(2460)^+ \rightarrow D_s^{*+}\pi^0$ . The heavy quark symmetry implies the same size of counter-term contributions for  $B_s$  system as in the case of charm mesons. Therefore, by adopting the result of lattice calculation that  $B_s$  mesons, part of the  $(0^+, 1^+)$  doublet, have masses below  $BK$  and  $BK^*$ , we are able to predict their partial decay widths.

The basic HM $\chi$ PT formalism is introduced in Section II. In Section III we calculate decay widths of the two body strong decays of positive parity  $B_s$  doublet  $(0^+, 1^+)$ . In Section IV, the calculation of the three body decay width  $B_s(1^+) \rightarrow B_s^0\pi\pi$  decay mode will be presented, while a short conclusion will be given in Section V.

## II. FRAMEWORK

In our analysis we rely on HM $\chi$ PT (see e.g.[25, 26]). This approach combines the heavy quark effective theory with the chiral perturbation theory and can be used to describe decays of mesons that are composed of one light and one heavy quark. The chiral perturbation theory works very well in the case where pseudoscalar mesons have low momenta. In the heavy meson limit, heavy mesons, pseudoscalar and vector, as well as scalar and axial, become degenerate. The negative parity states are described by the field  $H$ , while the positive parity states are entering in

the field  $S$ :

$$H = \frac{1}{2}(1 + v \cdot \gamma)[P_\mu^* \gamma^\mu - P \gamma_5], \quad S = \frac{1}{2}(1 + v \cdot \gamma)[D_\mu^* \gamma^\mu \gamma_5 - D], \quad (1)$$

where  $P_\mu^*$  and  $P$  annihilate the vector and pseudoscalar mesons respectively, while  $D_\mu^*$  and  $D$  annihilate the axial-vector and scalar mesons, respectively. Within chiral perturbation theory, the light pseudoscalar mesons are accommodated into the octet  $\Sigma = \xi^2 = e^{(2i\Pi/f)}$  with

$$\Pi = \begin{pmatrix} \pi^0/\sqrt{2} + \eta_8/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta_8/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta_8/\sqrt{6} \end{pmatrix} \quad (2)$$

and  $f \sim 120$  MeV at one loop level [27]. The leading order of the HM $\chi$ PT Lagrangian, that describes the interaction of heavy and light mesons, can be written as

$$\begin{aligned} \mathcal{L} = & -Tr[\bar{H}_a(iv \cdot \mathcal{D}_{ab} - \delta_{ab}\Delta_H)H_b] + gTr[\bar{H}_b H_a \gamma \cdot \mathcal{A}_{ab} \gamma_5] \\ & + Tr[\bar{S}_a(iv \cdot \mathcal{D}_{ab}) - \delta_{ab}\Delta_S]S_b + \tilde{g}Tr[\bar{S}_b S_a \gamma \cdot \mathcal{A}_{ab} \gamma_5] + hTr[\bar{H}_b S_a \gamma \cdot \mathcal{A}_{ab} \gamma_5], \end{aligned} \quad (3)$$

where  $\mathcal{D}_{ab}^\mu = \delta_{ab}\partial^\mu - \mathcal{V}_{ab}^\mu$  is a heavy meson covariant derivative,  $\mathcal{V}_\mu = 1/2(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger)$  is the light meson vector current and  $\mathcal{A}_\mu = i/2(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger)$  is the light meson axial current. A trace is taken over spin matrices and repeated light quark flavour indices. All terms in (3) are of the order  $\mathcal{O}(p)$  in the chiral power counting (see e.g. [22]). Following notation of [5],  $\Delta_{SH} = \Delta_S - \Delta_H = 375$  GeV and in order to maintain well behaved chiral expansion, we consider that this difference is of the order of pion momentum,  $\Delta_{SH} \sim \mathcal{O}(p)$  as in [22].

Light mesons are described by the Lagrangian [25, 26], which is of the order  $\mathcal{O}(p^2)$  in the chiral expansion

$$\mathcal{L}_0 = \frac{f^2}{8}Tr[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger] + \frac{f^2 \lambda_0}{4}Tr[m_q^\xi \Sigma + \Sigma m_q^\xi], \quad (4)$$

with the  $\lambda_0 = m_\pi^2/(m_u + m_d) = (m_{K^+}^2 - m_{K^0}^2)/(m_u + m_d) = (m_K^2 - m_\pi^2/2)/m_s$ . From the second term in (4), we can derive the  $\eta - \pi$  mixing Lagrangian [28, 29]:

$$\mathcal{L}_{\eta-\pi_0} = \frac{m_\pi^2(m_u - m_d)}{\sqrt{3}(m_u + m_d)}\pi_0\eta. \quad (5)$$

The scalar (pseudoscalar) and vector (axial-vector) heavy meson propagators can be written in the form:

$$\frac{i}{2(k \cdot v - \Delta_i)} \quad \text{and} \quad \frac{-i(g^{\mu\nu} - v^\mu v^\nu)}{2(k \cdot v - \Delta_i)} \quad (6)$$

respectively, where  $\Delta_i$  in the propagator represents the residual mass of the corresponding field. Residual masses are responsible for mass splitting of heavy meson states. The difference  $\Delta_{SH}$  splits the masses of positive and negative parity states. In addition, we also have a mass splitting between  $B_s$  and  $B$  states as well as a mass splitting between vector (axialvector) and pseudoscalar (scalar) fields. According to [30], the mass splitting between  $B_s$  and  $B$  states is 87 MeV, while the splitting between vector and pseudoscalar states is 45 MeV. Since these splittings are much smaller than  $\Delta_{SH}$ , they can be safely neglected.

The coupling constants  $g$ ,  $h$  and  $\tilde{g}$  were already discussed by several authors and determined by several methods [31]–[44]. We will use recent results of the lattice QCD:  $g = 0.54(3)(^{+2}_{-4})$  [36],  $\tilde{g} = -0.122(8)(6)$  and  $h = 0.84(3)(2)$  [42]. The lattice results will be also used for the  $B_{s0}^*$  and  $B_{s1}$  masses, as well as  $\Delta_{SH}$  [5]:  $m_{B_{s0}} = 5,711(13)(19)$  GeV,  $m_{B_{s1}} = 5.75(17)(19)$  GeV and  $\Delta_{SH} = 375(13)(19)$  MeV.

In order to absorb divergences coming from loop integrals, one needs to include counter-terms. Following [21, 22] counter-term Lagrangian can be written as:

$$\begin{aligned} \mathcal{L}_{ct} = & \lambda_1[\bar{H}_b \bar{H}_a(m_q^\xi)_{ba}] + \lambda'_1[\bar{H}_a \bar{H}_a(m_q^\xi)_{bb}] - \tilde{\lambda}_1[\bar{S}_b \bar{S}_a(m_q^\xi)_{ba}] - \tilde{\lambda}'_1[\bar{S}_a \bar{S}_a(m_q^\xi)_{bb}] + \\ & \frac{h\kappa'_1\lambda_0}{(4\pi f)^2}Tr[(\bar{H}S\gamma_\mu \mathcal{A}^\mu \gamma_5)_{ab}(m_q^\xi)_{ba}] + \frac{h\kappa'_3\lambda_0}{(4\pi f)^2}Tr[(\bar{H}S\gamma_\mu \mathcal{A}^\mu \gamma_5)_{aa}(m_q^\xi)_{bb}] + \end{aligned}$$

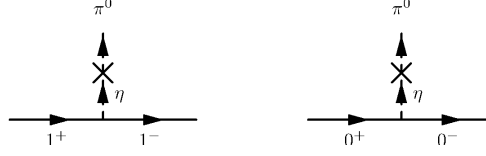


Figure 1. Tree level contribution to  $B_{s0}^{*0} \rightarrow B_s \pi^0$  and  $B_{s1}^0 \rightarrow B_s^* \pi^0$  decay modes.

$$\begin{aligned} & \frac{h\kappa'_5\lambda_0}{(4\pi f)^2} \text{Tr}[\bar{H}_a S_a \gamma_\mu \mathcal{A}_{bc}^\mu \gamma_5 (m_q^\xi)_{cb}] + \frac{h\kappa'_9\lambda_0}{(4\pi f)^2} \text{Tr}[\bar{H}_c S_a \gamma_\mu \mathcal{A}_{bc}^\mu \gamma_5 (m_q^\xi)_{ab}] + \\ & \frac{\delta'_2}{(4\pi f)^2} \text{Tr}[\bar{H}_a S_b i v \cdot \mathcal{D}_{bc} \gamma_\mu \mathcal{A}_{ca}^\mu \gamma_5] + \frac{\delta'_3}{(4\pi f)^2} \text{Tr}[\bar{H}_a S_b i \gamma_\mu \cdot \mathcal{D}_{bc}^\mu v \cdot \mathcal{A}_{ca}^\mu \gamma_5] + h.c. + \dots, \end{aligned} \quad (7)$$

where  $m^\xi = (\xi m_q \xi - \xi^\dagger m_q \xi^\dagger)/2$  and  $D_{bc}^\alpha A_{ca}^\beta = \partial^\alpha A_{ba}^\beta + [v^\alpha A^\beta]_{ba}$ . At the given scale, the finite part of  $\kappa'_3$  can be absorbed into the definition of  $h$ . Parameters  $\lambda'_1$  and  $\tilde{\lambda}'_1$  can be absorbed into the definition of heavy meson masses by phase redefinition of  $H$  and  $S$ , while  $\lambda_1$  and  $\tilde{\lambda}_1$  split the masses of SU(3) flavor triplets of  $H_a$  and  $S_a$  [21, 22]. Therefore, only contributions proportional to  $\kappa'_1$ ,  $\kappa'_9$ ,  $\kappa'_5$ ,  $\delta'_2$  and  $\delta'_3$  will be explicitly included in the amplitudes.

### III. THE AMPLITUDES AND THE DECAY WIDTHS OF TWO BODY DECAY MODES

At the tree level, the  $B_{s0}^{*0} \rightarrow B_s \pi^0$  and  $B_{s1}^0 \rightarrow B_s^* \pi^0$  decays occur through  $\eta - \pi$  mixing as shown in Fig. 1. The decay widths can be written as:

$$\Gamma = \frac{h^2}{2\pi f^2} |k_\pi| E_\pi^2 \delta_{mix}^2, \quad (8)$$

where  $E_\pi$  and  $k_\pi$  are the energy and momenta of the outgoing pion and  $\delta_{mix}$  is the  $\eta - \pi$  mixing angle

$$\delta_{mix} = \frac{1}{2\sqrt{2}} \frac{m_u - m_d}{m_s - (m_u + m_d)/2} = \frac{-1}{87\sqrt{2}}. \quad (9)$$

This yields:

$$\Gamma(B_{s1}^0 \rightarrow B_s^* \pi^0) = 16 \text{ keV}, \quad \Gamma(B_{s0}^{*0} \rightarrow B_s \pi^0) = 18 \text{ keV}. \quad (10)$$

By including chiral loop corrections, the decay width can be rewritten as:

$$\Gamma = \frac{h^2}{2\pi f^2} |k_\pi| E_\pi^2 \delta_{mix}^2 \left| \frac{\sqrt{Z_{w,f} Z_{w,i}}}{Z_v} \right|^2, \quad (11)$$

where  $Z_{w,f}$  and  $Z_{w,i}$  denote wave function renormalization of the initial and final heavy meson states and  $Z_v$  represents the vertex corrections. The wave function renormalization factor is defined as

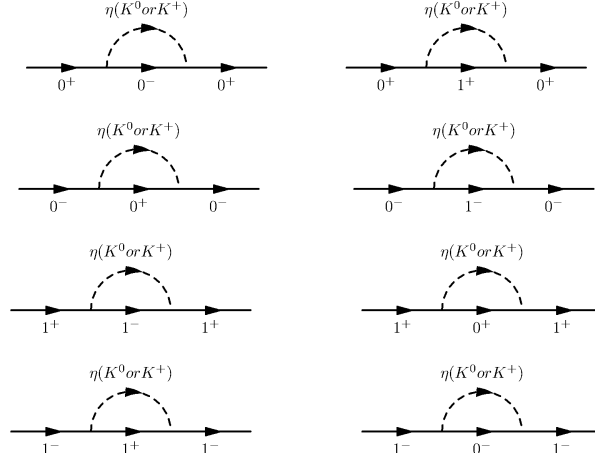
$$Z_{w,j} = 1 - \frac{1}{2} \frac{\partial \Pi_j(v \cdot p)}{\partial v \cdot p} \Big|_{on \text{ mass shell}}, \quad (12)$$

where  $\Pi_j(v \cdot p)$  is the meson self-energy calculated from the sunrise type diagrams in Fig 2. For  $Z_{w,j}$  we derive

$$Z_{w,j} = 1 - \mathcal{W}_j(m_{K^+}) - \mathcal{W}_j(m_{K^0}) - \frac{2}{3} \mathcal{W}_j(m_\eta), \quad (13)$$

with

$$\mathcal{W}_j(m_i) = \frac{1}{16\pi^2 f^2} (3\tilde{g}^2 \bar{B}'_{00}(0, m_i) - h^2 \bar{B}'_2(-\Delta_{SH}, -\Delta_{SH}, m_i)), \quad (14)$$

Figure 2. Chiral corrections to the  $B$  mesons wave functions.

for the positive parity mesons and

$$\mathcal{W}_j(m_i) = \frac{1}{16\pi^2 f^2} (3g^2 \bar{B}'_{00}(0, m_i) - h^2 \bar{B}'_2(\Delta_{SH}, \Delta_{SH}, m_i)) , \quad (15)$$

for the negative parity mesons. Here,  $\bar{B}'_{00}$ ,  $\bar{B}'_2$  are Passarino-Veltman loop integrals defined in Appendix A.

The vertex correction is defined as:

$$Z_v = 1 - \frac{\hat{\Gamma}(v \cdot p_i, v \cdot p_f, k^2)}{\hat{\Gamma}_0(v \cdot p_i, v \cdot p_f, k^2)} \Big|_{on\ mass\ shell} , \quad (16)$$

Here  $\hat{\Gamma}$  is the vertex amplitude calculated from the Feynman diagrams presented in Figs. 3 and 4, while  $\hat{\Gamma}_0$  is the vertex amplitude resulting from the tree level Feynman diagram (see Fig. 1):

$$Z_v = 1 - \left( \delta'_{mix} + \frac{2}{3} \mathcal{V}'(m_\eta) - \frac{1}{2} (\mathcal{V}(m_{K^+}) + \mathcal{V}(m_{K^0})) + \frac{1}{\sqrt{2}\delta_{mix}} (\mathcal{V}(m_{K^+}) - \mathcal{V}(m_{K^0})) + \mathcal{V}_{ct} \right) , \quad (17)$$

where  $\delta'_{mix} = 0.11$  includes corrections to the  $\eta - \pi$  mixing angle beyond tree level [21, 27], while  $\mathcal{V}$  and  $\mathcal{V}'$  are

$$\begin{aligned} \mathcal{V}(m_i) = & \frac{1}{16\pi^2 f^2} ((\bar{B}_{00}(-\Delta_{SH}, m_i) - \bar{B}_{00}(\Delta_{SH}, m_i) + \bar{B}_{11}(-\Delta_{SH}, m_i) - \bar{B}_{11}(\Delta_{SH}, m_i) \\ & - \Delta_{SH} \bar{B}_1(-\Delta_{SH}, m_i) - \Delta_{SH} \bar{B}_1(\Delta_{SH}, m_i))/2 \\ & - h^2 (\bar{B}'_{00}(-\Delta_{SH}, \Delta_{SH}, m_i) + \bar{B}'_{11}(-\Delta_{SH}, \Delta_{SH}, m_i)) + 3g\tilde{g}\bar{B}'_{00}(0, 0, m_i)) , \end{aligned} \quad (18)$$

$$\mathcal{V}'(m_i) = \frac{1}{16\pi^2 f^2} (-h^2 (\bar{B}'_{00}(-\Delta_{SH}, \Delta_{SH}, m_i) + \bar{B}'_{11}(-\Delta_{SH}, \Delta_{SH}, m_i)) + 3g\tilde{g}\bar{B}'_{00}(0, 0, m_i)) . \quad (19)$$

Note that the isospin violating nature of both decay amplitudes manifests itself either by the proportionality of amplitude to the mixing parameter  $\delta_{mix}$ , or by the mass difference  $m_{K^0} - m_{K^+}$ . Obviously in the isospin limit, amplitudes vanish for  $\delta_{mix} \rightarrow 0$  and  $m_{K^0} = m_{K^+}$ .

The finite parts of the counter-terms are collected in the term  $\mathcal{V}_{ct}$ :

$$\mathcal{V}_{ct} = \frac{1}{32\pi^2 f^2} \left( \left( m_K^2 - \frac{m_\pi^2}{2} \right) (\kappa'_1 + \kappa'_9) + \left( m_K^2 - m_\pi^2 + \frac{\sqrt{2}(m_{K^+}^2 - m_{K^0}^2)}{\delta_{mix}} \right) \kappa'_5 + \frac{E_\pi}{2\lambda_0} (\delta'_2 + \delta'_3) \right) . \quad (20)$$

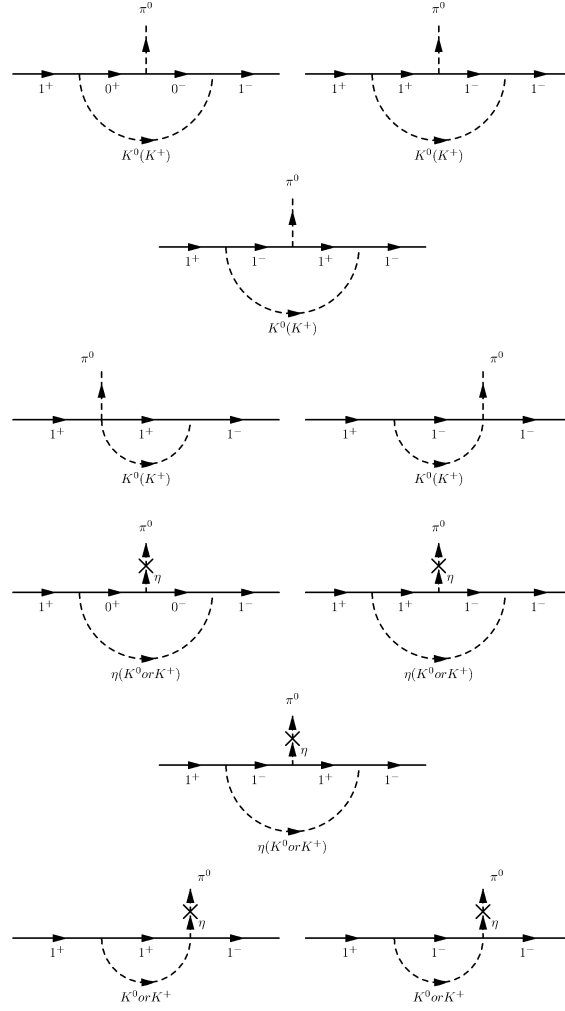


Figure 3. Chiral corrections to the  $B_{s1}^0 \rightarrow B_s^* \pi^0$  decay mode.

Neglecting the terms that are multiplied by  $m_\pi^2$  and  $\frac{E_\pi}{2\lambda_0}$  and by taking  $m_{K^+}^2 = m_{K^0}^2$ , all counter-terms can be replaced with the linear combination  $\kappa' = \kappa'_1 + \kappa'_9 + \kappa'_5$ , yielding:

$$\mathcal{V}_{ct} = \frac{m_K^2}{32\pi^2 f^2} \kappa'. \quad (21)$$

Due to heavy meson symmetry, the same counter-term appears also in the case of  $D_s$  positive parity meson decays. In [18] we were able to constrained the size of this counter-term using the experimentally known ratio of the decay widths of the  $D_{s1}(2460) \rightarrow D_s^* \pi$  and  $D_{s1}(2460) \rightarrow D_s \pi \pi$  decay modes. The decay widths are also rather sensitive to the value of the coupling constant  $h$  as already noticed in [18], for the charm meson decays. The wave-function renormalization factor is responsible for this behaviour. The dependence of the decay widths on the coupling constant  $h$  is shown in Fig. 5.

As seen from Fig. 5, the decay widths are in the range of  $(0.1 - 55) \text{ keV}$  for the range of coupling constant  $h = -0.84(3)(2)$  as found by lattice calculation [42]. Note that we use range of values for the counter-term  $(0.1 - 1.2)$  as found in [18]. For the central value  $h = 0.84$ , the range is  $1 \text{ keV} \leq \Gamma(B_{s1}^0 \rightarrow B_s^* \pi^0) \leq 30 \text{ keV}$ . The decay rates for  $B_{s1}^0 \rightarrow B_s^* \pi^0$  and  $B_{s0}^0 \rightarrow B_s \pi^0$  are almost equal, with the small difference due to the different masses of the final and initial  $B_s$  states.

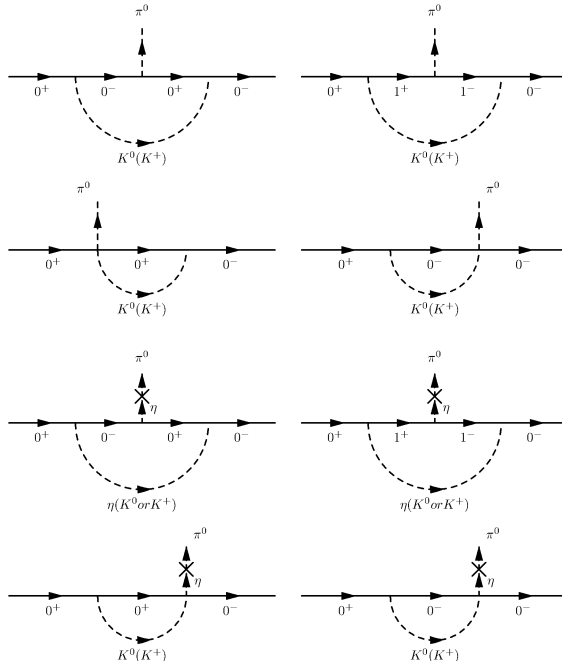


Figure 4. Chiral corrections to the  $B_s^{*0} \rightarrow B_s \pi^0$  decay mode.

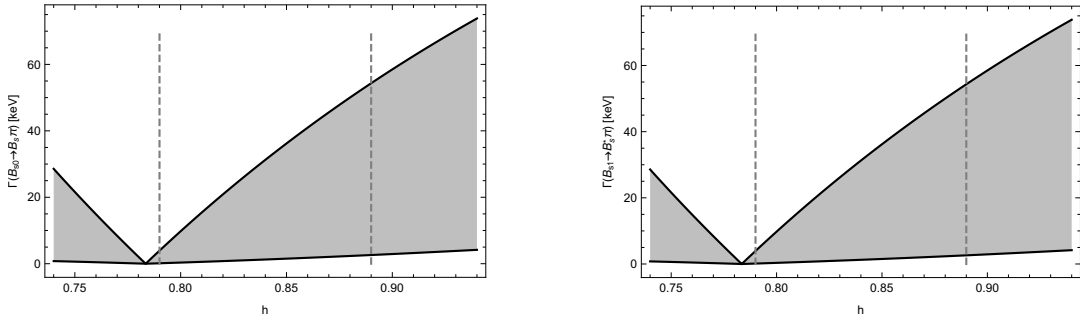


Figure 5. Dependence of  $\Gamma(B_{s1}^0 \rightarrow B_s \pi^0)$  (right) and  $\Gamma(B_{s0}^{*0} \rightarrow B_s \pi^0)$  (left) on the coupling constant  $h$ .

#### IV. THE THREE BODY DECAYS: AMPLITUDES AND DECAY WIDTHS

In the case of  $B_{s1}^0$ , a three body decays  $B_{s1}^0 \rightarrow B_s^0 \pi \pi$  are also possible. The  $B_{s1}^0 \rightarrow B_s^0 \pi \pi$  decay width, averaged over the  $B_{s1}^0$  polarisations, can be written as:

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32M_i^3} |\mathcal{M}|^2 dm_{12}^2 dm_{23}^2, \quad (22)$$

where  $M_i$  denotes the mass of  $B_{s1}^0$ . If  $p_-$  and  $p_+$  are the momenta of  $\pi^+$  and  $\pi^-$  respectively, and  $q$  is the momentum of  $D_s^+$ , then  $dm_{12}^2 = (p_+ + p_-)^2$  and  $dm_{23}^2 = (p_- + q)^2$ . In the heavy quark limit  $P^\mu = M_i v^\mu$ ,  $q^\mu = M_f v^\mu$  and  $\epsilon \cdot v = 0$ , the amplitude is simplified to the following form:

$$\mathcal{M} = \mathcal{A} \epsilon \cdot (p_+ - p_-) = \mathcal{A} \epsilon \cdot \Delta p.$$

The non-vanishing Feynman diagrams that contribute to the amplitude  $\mathcal{A}$  are presented in Fig. 6. Note that all diagrams with  $\eta$  meson in the loop give vanishing contribution, as it was already discussed in [18]. The amplitude  $\mathcal{A}$ , can then be written as:

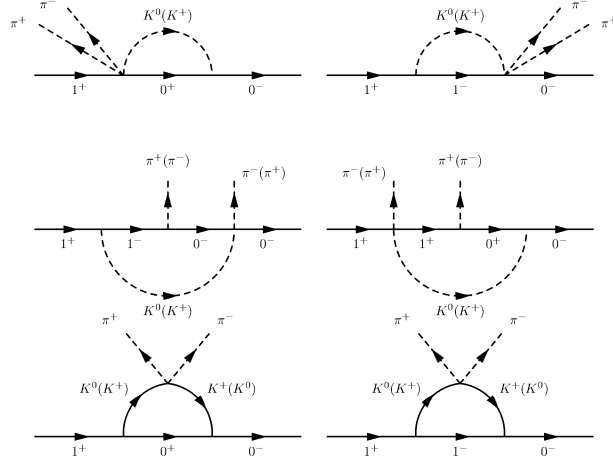


Figure 6. Non-vanishing contributions to  $B_{s1}^0 \rightarrow B_s^0 \pi^+ \pi^-$  decay amplitude.

$$\mathcal{A} = \frac{h\sqrt{M_i M_f}}{16\pi^2 f^4} (a_1 + a_2 + b_1 + b_2 + c_1 + c_2) , \quad (23)$$

where parts of the amplitudes can be written as a linear combinations of the Veltman-Pasarino functions:

$$a_1 = \frac{g}{2} (\bar{B}_1(-\Delta_{SH}, m_{K^0}) - \bar{B}_1(-\Delta_{SH}, m_{K^+})) , \quad (24)$$

$$a_2 = \frac{\tilde{g}}{2} (\bar{B}_1(\Delta_{SH}, m_{K^0}) - \bar{B}_1(\Delta_{SH}, m_{K^+})) , \quad (25)$$

$$b_1 = 2g \left( (\bar{B}_2'(-\Delta_{SH}, -\Delta_{SH}/2, m_{K^0}) - \Delta/2 \cdot \bar{B}_1'(-\Delta_{SH}, -\Delta_{SH}/2, m_{K^0})) \right. \\ \left. - (\bar{B}_2'(-\Delta_{SH}, -\Delta_{SH}/2, m_{K^+}) - \Delta/2 \cdot \bar{B}_1'(-\Delta_{SH}, -\Delta_{SH}/2, m_{K^+})) \right) , \quad (26)$$

$$b_2 = 2\tilde{g} \left( (\bar{B}_2'(\Delta_{SH}/2, \Delta_{SH}, m_{K^0}) + \Delta/2 \cdot \bar{B}_1'(\Delta_{SH}/2, \Delta_{SH}, m_{K^0})) \right. \\ \left. - (\bar{B}_2'(\Delta_{SH}/2, \Delta_{SH}, m_{K^+}) + \Delta/2 \cdot \bar{B}_1'(\Delta_{SH}/2, \Delta_{SH}, m_{K^+})) \right) , \quad (27)$$

$$c_1 = -2g \left( (B_{00}(m_{K^0}) - \Delta_{SH} \bar{C}_{00}(-\Delta_{SH}, m_{K^0})) - (B_{00}(m_{K^+}) - \Delta_{SH} \bar{C}_{00}(-\Delta_{SH}, m_{K^+})) \right)$$

$$c_2 = -2\tilde{g} (B_{00}(m_{K^0}) - B_{00}(m_{K^+})) . \quad (28)$$

Here,  $\bar{B}_1$ ,  $\bar{B}_2$ ,  $B_{00}$  and  $\bar{C}_{00}$  are the Passarino - Veltman loop integrals defined in Appendix A. As the  $B_{s1}^0 \rightarrow B_s^0 \pi^+ \pi^-$  decay mode does not have any tree level contributions from heavy meson Lagrangian, the amplitude is expected to be finite. Although some of the above integrals are divergent, this divergences cancel out as expected, when we take the sum of all contributions. We can also notice, that the amplitude vanishes in the case of  $m_{K^+} = m_{K^0}$ , showing the nature of isospin violating decay mode. The obtained decay widths are:

$$\Gamma(B_{s1}^0 \rightarrow B_s^0 \pi^+ \pi^-) = (1 \pm 0.3) \times 10^{-3} \text{ keV} , \quad \Gamma(B_{s1}^0 \rightarrow B_s^0 \pi^0 \pi^0) = (0.7 \pm 0.2) \times 10^{-3} \text{ keV} ,$$

In the case of  $B_{s1}^0 \rightarrow B_s^0 \pi^0 \pi^0$  a factor 1/2 was taken into account due to two identical mesons in the final state.



method	$\Gamma(B_{s0}^{*0} \rightarrow B_s \pi^0)$ [keV]	$\Gamma(B_{s1}^0 \rightarrow B_s^* \pi^0)$ [keV]	$\Gamma(B_{s1}^0 \rightarrow B_s \pi \pi)$ [keV]
molecule picture [20]	46.7	50.1	
heavy quark and chiral symmetry [13]	21.5	21.5	$\approx 0.05$
heavy chiral unitary approach [11, 12]	7.92	10.36	
$^3P_0$ model [19]	35	38	
chiral loop corrections (this work)	$< 55$	$< 50$	$\approx 0.001$

Table I. Predictions of the  $B_{s0}^{*0} \rightarrow B_s \pi^0$  and  $B_{s1}^0 \rightarrow B_s^* \pi^0$  decay widths.

## V. COMMENTS AND DISCUSSION

Using systematically  $\text{HM}\chi\text{PT}$ , we determine the decay widths of the isospin violating decay modes of positive parity  $B_s$  mesons:  $B_s(0^+) \rightarrow B_s^0 \pi^0$ ,  $B_s(1^+) \rightarrow B_s^{*0} \pi^0$  and  $B_s(1^+) \rightarrow B_s^0 \pi \pi$ . Masses of the decaying particles and the values of the coupling constants are taken from the lattice studies.

We find that the decay width  $\Gamma(B_s(1^+) \rightarrow B_s^0 \pi \pi) \sim 10^{-3} \text{ keV}$ . This process occurs only at loop level and the decay amplitudes are proportional to the mass difference of  $K^+$  and  $K^0$ . The small available phase space additionally suppresses the decay width. This decay might be also approached by the exchange of the  $f_0$  resonances  $B_s(1^+) \rightarrow B_s^0 f_0 \rightarrow B_s^0 \pi \pi$  [13]. However, in the  $\text{HM}\chi\text{PT}$  this is a higher order contribution and therefore is not considered in our analysis. The approach of Ref. [13] (see Table I) uses the exchange of  $\sigma$  resonance in which there is a significant  $\bar{s}s$  component. However, recent lattice calculation of [49] disfavours such a content of  $\sigma$ .

The two body decays of  $B_{s0}^{*0}$  and  $B_{s1}^{*0}$  occur at three level through  $\eta - \pi$  mixing. We find that the chiral loop corrections can significantly enhance or suppress decay amplitudes being almost of the same order of magnitude as the three level contribution. We can only give a range of values for the decay widths. Namely, the decay widths are very sensitive to the value of the coupling constant  $h$  and change significantly if the coupling constant  $h$  is varied within the error bars determined by the lattice studies [42]. Also, the counter-terms are known only within a range of values in [18]. In Table I, we give results of other existing studies. Authors of [20] find higher values of decay widths in the molecular picture of positive parity  $B_s$  states. In their approach, however, wave function renormalization, which in our case tends to lower the decay widths significantly, is not taken into consideration. Note also that contributions of  $K^*$  loops, present in [20], are a higher order correction in  $\text{HM}\chi\text{PT}$  approach and therefore not included in our analysis.

It will be interesting if current experimental searches at LHCb and planned studies at Belle II would lead to discovery of both states  $B_{s0}^{*0}$  and  $B_{s1}^{*0}$ . We hope that our study might shed more light on this issue.

## ACKNOWLEDGMENTS

The work of SF was supported in part by the Slovenian Research Agency.

## Appendix A: Loop integrals

By employing dimensional regularization, in the renormalization scheme with  $\delta = \frac{2}{4-D} - \gamma_E + \ln 4\pi + 1 = 0$ , we have:

$$A_0(m) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{d^D k}{(k^2 - m^2 + i\epsilon)} = m^2 \left( \delta - \ln \frac{m^2}{\mu^2} \right) + \mathcal{O}(D-4),$$

$$\begin{aligned} B_0(p, m, m) &= \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{d^D k}{(k^2 - m^2 + i\epsilon)((k+p)^2 - m^2 + i\epsilon)} \\ &= \delta - \int_0^1 \ln \frac{x^2 p^2 - x p^2 + m^2}{\mu^2} + \mathcal{O}(D-4), \end{aligned}$$

$$B_{00}(p, m, m) = \frac{1}{2(D-1)} [A_0(m) + (2m^2 - p^2/2)B_0(p, m, m)],$$

which in  $D \rightarrow 4$  limit gives

$$B_{00}(p, m, m) = \frac{1}{6} [A_0(m) + (2m^2 - p^2/2)B_0(p, m, m) + 2m^2 - p^2/3],$$

$$B_{00}(m) = B_{00}(\Delta_M v, m, m).$$

Loop integrals with one heavy meson propagator are:

$$\begin{aligned} \bar{B}_0(\Delta, m) &= \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{d^D k}{(k^2 - m^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)} = \\ &-2\Delta \left[ \delta - \ln \frac{m^2}{\mu^2} - 2F\left(\frac{m}{\Delta}\right) + 1 \right] + \mathcal{O}(D-4), \end{aligned}$$

with

$$F(1/x) = \begin{cases} \frac{1}{x} \sqrt{x^2 - 1} \ln(x + \sqrt{x^2 - 1} + i\epsilon); & |x| > 1, \\ \frac{-1}{x} \sqrt{1 - x^2} \left( \frac{\pi}{2} - \tan^{-1} \left( \frac{x}{\sqrt{1-x^2}} \right) \right); & |x| \leq 1, \end{cases}$$

$$\bar{B}^\mu(\Delta, m) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{k^\mu d^D k}{(k^2 - m^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)} = \bar{B}_1(\Delta, m)v^\mu,$$

$$\bar{B}_1(\Delta, m) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{k \cdot v d^D k}{(k^2 - m^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)} = A_0(m) + \Delta \bar{B}_0(\Delta, m),$$

$$\bar{B}^{\mu\nu}(\Delta, m) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{k^\mu k^\nu d^D k}{(k^2 - m^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)} = \bar{B}_{00}(\Delta, m)g^{\mu\nu} + \bar{B}_{11}(\Delta, m)v^\mu v^\nu,$$

$$\bar{B}_{00}(\Delta, m) = \frac{1}{D-1} [(m^2 - \Delta^2)\bar{B}_0(\Delta, m) - \Delta A_0(m)],$$

which in  $D \rightarrow 4$  gives

$$\bar{B}_{00}(\Delta, m) = \frac{1}{3} [(m^2 - \Delta^2)\bar{B}_0(\Delta, m) - \Delta A_0(m) + 2\Delta/3(3m^2 - 2\Delta^2)],$$

$$\bar{B}_{11}(\Delta, m) = \frac{1}{D-1} [(D\Delta^2 - m^2)\bar{B}_0(\Delta, m) + D\Delta A_0(m)],$$

which in  $D \rightarrow 4$  gives

$$\bar{B}_{11}(\Delta, m) = \frac{1}{3} [(4\Delta^2 - m^2)\bar{B}_0(\Delta, m) + 4\Delta A_0(m) - 2\Delta/3(3m^2 - 2\Delta^2)],$$

$$\bar{B}_2(\Delta, m) = \bar{B}_{00}(\Delta, m) + \bar{B}_{11}(\Delta, m),$$

$$\bar{B}'_0(\Delta_1, \Delta_2, m) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{d^D k}{(k^2 - m^2)(v \cdot k - \Delta_1)(v \cdot k - \Delta_2)} = \frac{1}{\Delta_1 - \Delta_2} [\bar{B}_0(\Delta_1, m) - \bar{B}_0(\Delta_2, m)],$$

$$\bar{B}^{\mu\nu}(\Delta_1, \Delta_2, m) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{k^\mu d^D k}{(k^2 - m^2)(v \cdot k - \Delta_1)(v \cdot k - \Delta_2)} = \bar{B}'_1(\Delta_1, \Delta_2, m)v^\mu,$$

$$\bar{B}'_1(\Delta_1, \Delta_2, m) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{k \cdot v d^D k}{(k^2 - m^2)(v \cdot k - \Delta_1)(v \cdot k - \Delta_2)} = \bar{B}_0(\Delta_2, m) + \Delta_1 \bar{B}'_0(\Delta_1, \Delta_2, m),$$

$$\bar{B}'_2(\Delta_1, \Delta_2, m) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{(k \cdot v)^2 d^D k}{(k^2 - m^2)(v \cdot k - \Delta_1)(v \cdot k - \Delta_2)} =$$

$$A_0(m) + (\Delta_1 + \Delta_2) \bar{B}_0(\Delta_2, m) + \Delta_1^2 \bar{B}'_0(\Delta_1, \Delta_2, m),$$

$$\bar{B}^{\mu\nu'}(\Delta_1, \Delta_2, m) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{k^\mu k^\nu d^D k}{(k^2 - m^2)(v \cdot k - \Delta_1)(v \cdot k - \Delta_2)} =$$

$$\bar{B}'_{00}(\Delta_1, \Delta_2, m) g^{\mu\nu} + \bar{B}'_{11}(\Delta_1, \Delta_2, m) v^\mu v^\nu,$$

$$\bar{B}'_{00}(\Delta_1, \Delta_2, m) = \frac{1}{D-1} [m^2 \bar{B}'_0(\Delta_1, \Delta_2, m) - \Delta_1 \bar{B}'_1(\Delta_1, \Delta_2, m) - \bar{B}_1(\Delta_2, m)],$$

which in  $D \rightarrow 4$  gives

$$\frac{1}{3} [m^2 \bar{B}'_0(\Delta_1, \Delta_2, m) - \Delta_1 \bar{B}'_1(\Delta_1, \Delta_2, m) - \bar{B}_1(\Delta_2, m) + 2/3(3m^2 - 2(\Delta_1^2 + \Delta_2^2 + \Delta_1 \Delta_2))],$$

$$\bar{B}'_{11}(\Delta_1, \Delta_2, m) = \frac{1}{D-1} [-m^2 \bar{B}'_0(\Delta_1, \Delta_2, m) + D \Delta_1 \bar{B}'_1(\Delta_1, \Delta_2, m) + D \bar{B}_1(\Delta_2, m)],$$

which in  $D \rightarrow 4$  gives

$$\frac{1}{3} [-m^2 \bar{B}'_0(\Delta_1, \Delta_2, m) + 4 \Delta_1 \bar{B}'_1(\Delta_1, \Delta_2, m) + 4 \bar{B}_1(\Delta_2, m) - 2/3(3m^2 - 2(\Delta_1^2 + \Delta_2^2 + \Delta_1 \Delta_2))],$$

Loop integrals with two heavy meson propagator are:

$$\bar{C}^\mu(p, \Delta, m_1, m_2) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{k^\mu d^D k}{(k^2 - m_1^2 + i\epsilon)((k-p)^2 - m_2^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)} =$$

$$\bar{C}_1(p, \Delta, m_1, m_2) v^\mu,$$

$$\bar{C}_1(p, \Delta, m_1, m_2) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{k \cdot v d^D k}{(k^2 - m_1^2 + i\epsilon)((k-p)^2 - m_2^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)} =$$

$$B_0(p, m_1, m_2) + \Delta \bar{C}_0(p, \Delta, m_1, m_2),$$

$$\bar{C}^{\mu\nu}(p, \Delta, m_1, m_2) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{k^\mu k^\nu d^D k}{(k^2 - m_1^2 + i\epsilon)((k-p)^2 - m_2^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)} =$$

$$\bar{C}_{00}(p, \Delta, m_1, m_2) g^{\mu\nu} + \bar{C}_{11}(p, \Delta, m_1, m_2) v^\mu v^\nu,$$

$$\bar{C}_{00}(\Delta, m) = \bar{C}_{00}(-\Delta_M v, \Delta, m, m) = \frac{1}{D-1} [\bar{B}_0(-\Delta_M + \Delta, m) - (\Delta_M/2 + \Delta) B_0(\Delta_M v, m, m) + (m^2 - \Delta^2) \bar{C}_0(\Delta_M v, \Delta, m, m)],$$

which in  $D \rightarrow 4$  gives

$$\bar{C}_{00}(\Delta, m) = \bar{C}_{00}(-\Delta_M v, \Delta, m, m) = \frac{1}{3} [\bar{B}_0(-\Delta_M + \Delta, m) - (\Delta_M/2 + \Delta) B_0(\Delta_M v, m, m) + (m^2 - \Delta^2) \bar{C}_0(\Delta_M v, \Delta, m, m) - 2/3(3/2\Delta_M - \Delta)],$$

$$\bar{C}_{11}(\Delta, m) = \bar{C}_{11}(-\Delta_M v, \Delta, m, m) = \frac{1}{D-1} [-\bar{B}_0(-\Delta_M + \Delta, m) + D(\Delta_M/2 + \Delta) B_0(\Delta_M v, m, m) - (m^2 - D\Delta^2) \bar{C}_0(\Delta_M v, \Delta, m, m)],$$

which in  $D \rightarrow 4$  gives

$$\bar{C}_{11}(\Delta, m) = \bar{C}_{11}(-\Delta_M v, \Delta, m, m) = \frac{1}{3} [-\bar{B}_0(-\Delta_M + \Delta, m) + 4(\Delta_M/2 + \Delta) B_0(\Delta_M v, m, m) - (m^2 - 4\Delta^2) \bar{C}_0(\Delta_M v, \Delta, m, m) + 2/3(3/2\Delta_M - \Delta)].$$

The the calculation of the integral:

$$\bar{C}_0(p, \Delta, m_1, m_2) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{d^D k}{(k^2 - m_1^2 + i\epsilon)((k-p)^2 - m_2^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)}$$

is done in [50]. For some calculations, we used the program FeynCalc [51].

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